# OPTIMAL LOAD SCHEDULING OF HYDRO - THERMAL SYSTEM INCLUDING TRANSMISSION LOSSES

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

*by*

MAJOR S. C. DEY

*to the*

**DEPARTMENT OF ELECTRICAL ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

**APRIL, 1987**

EE-1987-M-DEY-OPT

DEDICATED TO

MY PARENTS

# CERTIFICATE

Certified that this work entitled 'OPTIMAL LOAD SCHEDULING OF HYDRO-THERMAL SYSTEM INCLUDING TRANSMISSION LOSSES' by Major S.C. Dey has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Dr. L.P. SINGH
Professor
Department of Electrical Engineering
Indian Institute of Technology
Kanpur

## ACKNOWLEDGEMENT

It gives me immense pleasure to take this opportunity to express my gratitude with reverence to my thesis supervisor Dr. L.P. Singh, Professor, Electrical Engineering, who initiated me into the work reported in this thesis and provided with invaluable and unfailing guidance for carrying out this work. Without his help and encouragement, this work would never have reached a completion.

I would also like to express my indebtness to my friends, especially, Mr. Arun Singhal, Mr. V.P. Singh, Mr. Ajai Srivastava, Mr. Senthil and Major V.K. Bhatia for their constant help and co-operation at different stages of the work. The pains taken by Shri Yogendra for his skilful typing is gratefully acknowledged. Finally, I thank Mr. Quasim for preparing drawings of flowchart used in this work.

Major S.C. Dey

ABSTRACT

With the growth in demand of electricity for various purposes, today power system has become complex in nature with a number of interconnections of different generating stations located at different places and supplying composite loads connected to it at different voltage levels. The generating stations include both hydro as well as thermal plants connected in a grid. In such a system, if the installed capacity exceeds total demand plus losses, a power engineer will have a number of strategies to meet the load demand, but the best will be the one which results in the minimum operating cost satisfying all the operating constraints on the system. Several programming techniques have been used in the past to get an optimal solution. The present work as reported in this thesis exploits the technique of dynamic programming to arrive at a realistic answer in solving a short range scheduling problem of a hydro-thermal system including losses. Achievements of the present work are increased accuracy, faster rate of convergence, reduction in computational efforts and simple algorithm with an easy mathematical approach.

# CONTENTS


| | | | Page |
|---|---|---|---|
| CHAPTER | I | INTRODUCTION | 1 |
| CHAPTER | II | DYNAMIC PROGRAMMING | |
| | 2.1 | Introduction | 27 |
| | 2.2 | Multistage Decision Process | 27 |
| | 2.3 | Principle of Optimality | 31 |
| | 2.4 | Dynamic Programming Technique | 32 |
| CHAPTER | III | ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEMS | |
| | 3.1 | Introduction | 37 |
| | 3.2 | Lagrangian Multiplier Approach for Solution of Economic Load Scheduling of Hydro-Thermal System | 38 |
| CHAPTER | IV | OPTIMAL LOAD SCHEDULING OF HYDRO-THERMAL SYSTEM INCLUDING LOSSES | |
| | 4.1 | Introduction | 44 |
| | 4.2 | Problem Formulation Using Dynamic Programming | 45 |
| | 4.3 | Solution Procedure to Include Transmission Losses Using Dynamic Programming | 50 |
| | 4.4 | Case Study | 55 |
| CHAPTER | V | CONCLUSION | 59 |
| APPENDIX | | | 63 |
| REFERENCES | | | 66 |

# CHAPTER I

## INTRODUCTION

In the present day when economy at every walk of life is a basic requirement, the electric consumers will always look forward to the cheapest electrical energy made available to them. For a power system to return a reasonable profit on the capital invested, various modes of operation exist right from generation to distribution to arrive at an economic one; but of all, economic load scheduling is important and challenging. Rates fixed by regulatory bodies and the importance of conservation of fuel place extreme pressure on power companies to achieve maximum efficiency of operation and to improve it continuously in order to maintain a reasonable relation between cost of a Kwh to a consumer and the cost to the company of delivering a Kwh in the face of constantly rising prices of fuel, labour, supplies and maintenance. In operating the system for any load condition, the contribution from each plant must be determined so that the cost of delivered power is a minimum. This is the challenge before an engineer and the central theme of the present work is to meet this challenge limiting the considerations to the hydro-thermal system on a short-range basis.

In the earlier days, there used to be mostly small thermal plants for each locality known as urban systems with the sources close to the load centre. But with the considerable growth in the demand of electricity and rising cost of fuel and its limited reserve, a need was felt to include hydel plants also alongwith the thermal plants in a grid system with the basic aim to achieve minimum operating cost and exploit the capability of hydel plants having higher reliability and greater speed of response when run as a peak load plant. However, to achieve these, a number of constraints have to be satisfied. In an interconnected system, the generators may not be close to the load centre and hence, transmission losses will occur which have to be met to arrive at a realistic optimal solution resulting in minimum operating cost. The operating cost of thermal plants and also that of hydro-thermal plants is mainly the cost of fuel as the cost of water is negligible. Cost of labour, maintenance etc. are assumed to vary as a fixed percentage of fuel cost. Hence, cost function of hydro-thermal plant is defined as a nonlinear function of plant generations. For a short range problem, the level of water in the reservoir of hydel plant remains almost constant for a given time interval. Only constraint will be, therefore, the specified quantity of water to be utilised in each hydel plant in the given time interval. By economic or optimal load scheduling we mean to determine the gene-

rations of different plants, hydel as well as thermal, such that the total operating cost is optimal and various constraints as mentioned above are also satisfied. The problem, therefore, is the non-linear programming problem having nonlinear equality constraints.

Break through in the area of optimal generations of hydro-thermal system was probably first achieved by Chandler, Dandeno, Glimn and Kirchmayer [1]. They used Lagranges multiplier to convert the nonlinear objective function with the nonlinear equality constraints into an unconstrained objective function. The optimum scheduling problem as considered here is defined as follows:

While using specified amount of water from the hydro plants over a given period of time, it is desired to minimize the total fuel cost (in dollars). If it is assumed that, for the period of time under consideration, the hydro plants operate at essentially constant head, then the cordination equations whose solution gives the optimum schedule are

$$\frac{dF_n}{dP_{sn}} + \lambda \frac{\partial L_T}{\partial P_{sn}} = \lambda ; \quad n=1,2, \ldots \alpha \tag{1.1}$$

$$\gamma_j \frac{dw_j}{dPH_j} + \lambda \frac{\partial L_T}{\partial PH_j} = \lambda ; \quad j=\alpha+1,\alpha+2, \ldots \beta \tag{1.2}$$

where,

$\frac{dF_n}{dP_{sn}}$ = incremental fuel cost of thermal plant n in dollars per Mwh

$\frac{\partial L_T}{\partial P_{sn}}$ = incremental transmission loss of thermal plant n

$\frac{dw_j}{dPH_j}$ = incremental water discharge rate at hydro plant j in cubic feet per Mwh

$\frac{\partial L_T}{\partial PH_j}$ = incremental transmission loss of hydro plant j

$\lambda$ = incremental cost of received power in dollars per Mwh

$\gamma_j$ = constants which convert an incremental water discharge rate into an equivalent incremental plant cost.

$\alpha$ = number of thermal plants ; $w_j$ = water input to plant j in cubic feet per second

$\beta-\alpha$ = number of hydro plants

For the purpose of evaluating the savings possible by scheduling generation according to solution of co-ordination equations, the following methods of scheduling had been considered:

1. <u>The Co-ordination Equations Method</u>:

i.e. if the $(dF_n)/(dP_{sn})$ and $(dw_j)/(dPH_j)$ terms in equations (1.1) and (1.2) are assumed linear functions of output power, they can be written in slope intercept form as

$$\frac{dF_n}{dP_{sn}} = F_{nn}P_{sn}+f_n \tag{1.3}$$

and

$$\frac{dw_j}{dPH_j} = w_{jj}PH_j+w_j \tag{1.4}$$

By substituting the values of $(\frac{dF_n}{dP_{sn}})$ and $(\frac{dw_j}{dPH_j})$ from equation (1.3) and (1.4) into equations (1.1) and (1.2) respectively, the choice of $\lambda$ then determines a set of generations $P_{sn}$ and $PH_j$ and thus a particular received load.

2. <u>The Equal Incremental Plant Costs Method</u>:

If the terms $(\partial L_T)/(\partial P_{sn})$ and $(\partial L_T/\partial PH_j)$ in equations (1.1) and (1.2) are neglected, then again substituting the

value of $\frac{dF_n}{dP_{sn}}$ and $\frac{dw_j}{dPH_j}$ from (1.3)and (1.4) into (1.1) and (1.2), we get,

$$F_{nn}P_{sn}+f_n = \lambda ; \quad \gamma_j w_{jj} PH_j + \gamma w_j = \lambda$$

Again, the choice of $\lambda$ determines a set of generations $P_{sn}$ and $PH_j$ and thus a particular load.

3. <u>The maximum Efficiency Hydro Method</u> :

It is a proven practice to operate hydro plants near the point of maximum efficiency except during times of surplus water.

While working with these coordination equations on digital computers, Dandeno[2] observed that the direct application of these co-ordination equations give solutions which sometimes dictates generations outside the plant capacity and also negative generations of certain plants since restrictions regarding the plant capacities had not been included in the problem formulation.

However, significant improvement was achieved when Wije Perera[3] and others applied Pontryagin's Maximum Prin-

ciple followed by Kuhn-Tucker conditions to solve the problem imposing also the upper and lower bounds on the operating range of each plant in the form of inequality constraints.

Nonlinear programming technique was used by Sokkappa B.G.[4]. Here transmission losses were included by the use of loss coefficients. For each subinterval of optimisation, the constraint that is most likely to be violated, was picked up and a slack variable was associated with the constraint. The gradient of the objective function i.e. cost of the generation , was evaluated and the steepest descent method was used to obtain the solution of the problem, starting from a known initial solution i.e. generation schedule. However, this method was found to be inefficient when the system size becomes large.

Gopala Rao[5] solved a short range problem using Lasdon's decomposition technique[6] splitting the problem of higher dimensions into subproblem of smaller dimensions to be solved iteratively. The method suffers from a drawback i.e. the initial choice of the dual variable vector, which plays a significant role in the overall computational time, is crucial. A brief account of the method is given below.

The optimisation interval is subdivided into 'K' equal intervals such that the load demand during the interval at all the stations remains constant. To find transmission losses, A.C. load flow equations were used. The variables in the problem are bus voltage magnitudes and their angles. Maximum and minimum limits on active and reactive power generations and maximum limits on bus voltage magnitudes are imposed.

Let a system has 'M' hydro and 'N' thermal stations and 'm' total buses. Then, during $K^{th}$ interval, the injection equations can be expressed as,

$$I_i(\delta^K, E^K) = \sum_{j=1}^{m} E_i^K E_j^K Y_{ij} \cos(\theta_{ij} - \delta_i^K + \delta_j^K) \tag{1.5}$$

$$J_i(\delta^K, E^K) = - \sum_{j=1}^{m} E_i^K E_j^K Y_{ij} \sin(\theta_{ij} - \delta_i^K + \delta_j^K \tag{1.6}$$

where,

$E^K$ = bus voltage magnitude in $K^{th}$ interval

$\delta^K$ = bus voltage angle in $K^{th}$ interval

$Y_{ij}\angle\theta_{ij}$ = $ij^{th}$ element of Y-bus matrix

$I_i$, $J_i$ = injections into the system at $i^{th}$ bus.

Therefore, power generation equations can be written as

$$P_i^K(\delta^K, E^K) = I_i(\delta^K, E^K) + P_{Di}^K$$

$$Q_i^K(\delta^K, E^K) = J_i(\delta^K, E^K) + Q_{Di}^K$$

$$\text{for } i = 1, \ldots, (M+N)$$
$$\text{and} \quad K = 1, \ldots, K$$

where $P_i^K$ and $Q_i^K$ are active and reactive power generations at $i^{th}$ bus, and $P_{Di}^K$ and $Q_{Di}^K$ are active and reactive load demands at the same bus in the $K^{th}$ interval.

Similarly, the load demand equations will be,

$$\left.\begin{aligned} I_i(\delta^K, E^K) + P_{Di}^K &= 0 \\ J_i(\delta^K, E^K) + Q_{Di}^K &= 0 \end{aligned}\right\} \quad \begin{aligned} &\text{for } i = M+N+1 \ldots m \\ &\text{and } K = 1, \ldots, K \end{aligned}$$

The objective function, i.e. the cost of generation can be expressed as,

$$f(\delta^K, E^K) = \sum_{K=1}^{K} \sum_{i=M+1}^{M+N} C_i(P_i^K(\delta^K, E^K)) \tag{1.7}$$

Now, the optimisation problem can be mathematically stated as

Minimise $f(\delta^K, E^K)$
subject to the following constraints:

(i) Load Demand Equations,

$$I_i(\delta^K, E^K) + P^K_{Di} = 0$$

$$J_i(\delta^K, E^K) + Q^K_{Di} = 0$$

for $i = M+N+1, \ldots, m$
and $K = 1, \ldots, K$

(ii) operating limits on the system,

$$P_i^{min} \leq P_i^K(\delta^K, E^K) \leq P_i^{max}$$

$$Q_i^{min} \leq Q_i^K(\delta^K, E^K) \leq Q_i^{max}$$

$$E_i^K \leq E_i^{max} \quad ; \quad \text{for } i = 1, \ldots, M+N$$

and $K = 1, \ldots, K$

(iii) restrictions to use specified quantity of water,

$$\sum_{K=1}^{K} y_i(P_i^K(\delta^K, E^K)) = Y_i \qquad (1.8)$$

for $i = 1, \ldots, M$

where, $Y_i$ is the volume of water available for power generation

The objective function can, therefore, be written as,

$$f(\delta^K, E^K) = \sum_{K=1}^{K} \bar{f}_K (\delta^K, E^K) \tag{1.9}$$

where, $\bar{f}_K(\delta^K, E^K) = \sum_{i=M+1}^{M+N} C_i(P_i^K(\delta^K, E^K))$

Associating dual variables $u_1, \ldots, U_M$ with constraint equation (1.8), the Lagrangian function can be written as,

$$L(\delta^K, E^K, u) = \sum_{K=1}^{K} \bar{f}_K(\delta^K, E^K) + \sum_{i=1}^{M} u_i \left( \sum_{K=1}^{K} y_i(P_i^K(\delta^K, E^K)) - Y_i \right) \tag{1.10}$$

Therefore, we have a dual problem, i.e.

Maximise h(u) over all u

where $h(u) = \min_{\delta^K, E^K \in S^K} L\ (\delta^K, E^K, u)$ for $K = 1, \ldots, K$

The set $S^K$ also known as constraining set to which the $K^{th}$ sub problem variables $\delta^K$ and $E^K$ are to be constrained is

defined by constraints (i) & (ii) mentioned above.

Using Lasdon's decomposition technique[6], the Lagrangian function is minimised by solving the following 'K' sub problems, for K = 1, ..., K.

i.e. minimise $\bar{f}_K(\delta^K, E^K) + \sum_{i=1}^{M} u_i y_i (P_i^K(\delta^K, E^K))$

subject to the constraints (i) and (ii) mentioned above.

Therefore, we can say that the crux of the method mentioned above is to have an additively separable nonlinear programming problem split into subproblems and solved iteratively. The selection of dual variable $u_i$ is extremely crucial as the minimisation process is accomplished by it.

Hano, Tamura Y and Narita, S [7] have used continuous maximum principle of L.S. Pontryagin[13] to the most economic operation of Hydro-thermal systems. A brief account of the method employed by them is given below.

Let us consider, the system having two plants; one hydel and the other thermal, jointly supplying electric power to a load centre through a system having lossless transmission

lines. The rate of water in flow into the reservoir as J is assumed to be constant. Load demand is represented by the daily load curve spanning over 24 hours. The fuel cost of thermal plant is approximiated by the square function of the thermal output [i.e $F(G) = a_1 G + a_2 G^2$].

Let the water storage in the reservoir, the rate of water discharge, and rate of water inflow be expressed as $x_1(t)$, $u(t)$ and J respectively, then

$$\frac{dx_1}{dt} = J - u \tag{1.11}$$

It is assumed that the effect of evaporation, leakage, or overflow due to rainfall etc. have not been taken into account in equation (1.11) above. Turbine lower and upper bounds rates of water discharge are given by

$$\underline{u} \leq u \leq \bar{u} \tag{1.12}$$

The power output P of the hydro plant may be expressed approximately as

$$P = H_o \, (1 + C \, x_1) \, (u - q) \tag{1.13}$$

where, $H_o$ = basic water head

$C$ = correction factor of water head

$q$ = rate of non-effective water discharge

The power balance equation may be expressed as,

$$P_R = G + P \tag{1.14}$$

where, $P_R$ = the load demand

$P$ = Power output of hydel plant

$G$ = Output of thermal plant

Here, transmission losses have been neglected.

With these definitions, the problem of the economical load dispatching may be initiated in the simplest form as follows:

Given a time-dependent function of load as $P_R(t)$, and the initial and final storages of the water as $x_1(0)$ and $x_1(T)$, respectively, determine a control u such that the integrated fuel cost of the thermal plant

$$\phi = \int_0^T F(G)dt \tag{1.15}$$

be minimal, where $F(G)$ is the fuel cost.

Employing the maximum principle theorem, the problem will be restated as follows: Given a time dependent function $P_R(t)$, the initial and final states of the system as $x(0) = [x_1(0), 0]$ and $x(T) = [x_1(T), T]$, respectively and a set of differential equations governing the dynamics of the system as,

$$\frac{dx_o}{dt} = F(G) \tag{1.16}$$

$$\frac{dx_1}{dt} = J - u \tag{1.17}$$

$$\frac{dx_2}{dt} = 1, \tag{1.18}$$

determine the optimal control u which minimizes the integral function.

$$x_o(T) = \int_0^T F(G)dt \tag{1.19}$$

The optimizing condition of the maximum principle states, that, the optimal control here, is the one which maximizes the corresponding Hamiltonian function

$$H = - F(G) + \psi_1 \ (J - u) + \psi_2 \tag{1.20}$$

where, $\psi_1$ and $\psi_2$ are the adjoint variables to the system and are governed by a set of differential equations i.e.

$$\frac{d\psi_1}{dt} = \frac{-\partial H}{\partial x_1} = - \frac{dF}{dG} H_o C(u - q) \tag{1.21}$$

$$\frac{d\psi_2}{dt} = \frac{-\partial H}{\partial x_2} = - \frac{dF}{dG} \frac{dP_R(t)}{dt} \tag{1.22}$$

and also, $$\frac{\partial H}{\partial u} = \frac{-\partial F}{dG} \cdot \frac{\partial G}{\partial P} \frac{\partial P}{\partial u} - \psi_1 = \frac{dF}{dG} H_o(1 + Cx_1) - \psi_1 \tag{1.23}$$

making (1.23) equal to zero, we get

$$\frac{dF}{dG} = \frac{\psi_1}{H_o(1 + Cx_1)} \tag{1.24}$$

Since, G is a function of $u$ as is evident by equation (1.13) above, knowing $\frac{dF}{dG}$ from equation (1.24), G can be expressed as,

$$G = \frac{1}{2a_2} \left[\frac{\psi_1}{H_o(1+x_1)} - a_1\right] \tag{1.25}$$

If G lies outside its maximum and minimum values $\bar{G}$ and $\underline{G}$, modifications in the choice of $u$ is made to bring back G within the permissible region. From equation (1.17) and (1.19) we get,

$$u = \frac{P_R - G}{H_o(1+Cx_1)} + q \text{ if } \underline{u} \leq u \leq \bar{u}$$

$$= \bar{u} \qquad \text{if} \quad u > \bar{u}$$

$$= \underline{u} \qquad \text{if} \quad u < \underline{u}\,.$$

Since the maximum principle approach involves numerical integration of differential equations $\frac{dx_i}{dt} = J_i(x_{n+1}) - u_i$ ; $i = 1,2,\ldots,n$ ; $\frac{d\psi_i}{dt} = \frac{-\partial H}{\partial x_i}$ ; $i = 1,2, \ldots, n+1$, the time interval for each step of numerical integration should be small enough depending upon the degree of accuracy desired. As per the authors view, dynamic programming was found to be less efficient in terms of computation time and accuracy when compared to their proposed method. However, an effective method has not yet been developed to determine the initial values of the adjoint variables to a system given by equation (1.20) above. This is the serious drawback of the system and it has been accepted by the authors also.

Srivastava K.N. and Singh L.P.[8] have used the successive dual linear programming technique to obtain solutions to the complex problem of optimal scheduling of hydro-thermal systems including transmission losses for a short range problem. It has been claimed that the technique has succeeded in reduction of

storage and computational efforts. A brief description of the mathematical formulation of the work is presented below.

Let the cost characteristics of different thermal plants and discharge characteristics of hydel plants as a function of plant generations are represented by,

$$C_j(S_j^K) = a_j + b_j S_j^K + c_j (S_j^K)^2 \qquad (1.26)$$

and $$y_i(h_i^K) = p_i + q_i h_i^K + r_i (h_i^K)^2 \qquad (1.27)$$

where,

| | | |
|---|---|---|
| $S_j$ | = | Generation of thermal plant j in MW |
| $h_i$ | = | Generation of hydel plant i in MW |
| i | = | 1,..., M i.e. M hydel plants |
| j | = | 1,..., N i.e. N thermal plants |
| K | = | Sub intervals of optimization (usually a day descretized into K equal intervals) |
| $C_j(S_j^K)$ | = | cost function of $j^{th}$ thermal plant in Rs/hr |
| $y_i(h_i^K)$ | = | the discharge function of $i^{th}$ hydel plant in $m^3/hr$ |

$a_j, b_j, c_j, p_i, q_i, r_i$ are constants.

While considering the cost associated with a particular interval K, the subscript K is dropped from $S_j^K$ and $h_i^K$ and the objective function is written for that particular interval as follows :

$$f^K = \sum_{j=1}^{N} (a_j + b_j S_j + C_j S_j^2) + \sum_{i=1}^{M} \gamma_i (p_i + q_i h_i + r_i h_i^2) \qquad (1.28)$$

subject to constraints

$$\text{(a)} \quad \sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_D + P_L \, (S_j, h_i) \qquad (1.29)$$

$$\text{(b)} \quad S_j^{min} \leq S_j \leq S_j^{max} \qquad (1.30)$$

and $$\text{(c)} \quad h_i^{min} \leq h_i \leq h_i^{max} \qquad (1.31)$$

where, $P_D$ = demand in MW on the system

$P_L$ = transmission losses

Linearizing the scaler cost function of equation (1.28) by Taylor series expansion around the initial operating state $(S_j^o, h_i^o)$, the incremental cost function is obtained as,

$$\Delta f^K = \sum_{j=1}^{N} (b_j + 2C_j \, S_j^o) \, \Delta S_j + \sum_{i=1}^{M} \gamma_i (q_i + 2r_i h_i^o) \, \Delta h_i$$

i.e.

$$\Delta f^K = \sum_{i=1}^{M+N} d_i \ \Delta P_i \tag{1.32}$$

where $d_i$ = incremental cost co-efficient

$\Delta P_i$ = incremental change in the generation of plants.

The constraints of the equations (1.30),(1.31) and (1.29) can be rewritten as

$$g_i = S_i - S_i^{min} \geq 0 \tag{1.33}$$

$$g_{N+i} = S_i^{max} \geq 0 \quad \text{for} \quad i = 1, \ldots, N \tag{1.34}$$

$$g_{2N+M+i} = h_i^{max} - h_i \geq 0 \quad \text{for} \quad i = 1, \ldots, M \tag{1.35}$$

$$g_{2N+2M+1} = \sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i - P_D - P_L(S_j, h_i) = 0 \tag{1.36}$$

Linearizing the equations (1.33) to (1.36) around the current operating state by Taylor series expansion, the new linearized constraints are obtained as follows.

$$g_i = \Delta S_i \geq S_i^{min} - S_i^o \quad \text{for} \quad i = 1, \ldots, N \tag{1.37a}$$

$$g_{N+i} = -\Delta S_i \geq S_i^o - S_i^{max} \quad \text{for} \quad i = 1,\ldots, N \qquad (1.37b)$$

$$g_{2N+i} = \Delta h_i \geq h_i^{min} - h_i^o \quad \text{for} \quad i = 1,\ldots, M \qquad (1.38a)$$

$$g_{2N+M} = -\Delta h_i \geq h_i^o - S_i^{max} \quad \text{for } i = 1,\ldots,M \qquad (1.38b)$$

$$g_{2N+2M+1} = P_D + P_L(S_j^o, h_i^o) - \sum_{j=1}^{N} S_j^o - \sum_{i=1}^{M} h_i^o \qquad (1.39\_)$$

Equation (1.32), together with equation (1.37a) to (1.39_) forms a linear programming problem in incremental variables $\Delta S_j$ and $\Delta h_i$.

Using Dantzig's method to make problem variables to be non-negative, a non-negative vector Z of dimension (N+M) with elements $z_i$ is defined as follows,

$$z_i = \Delta S_j - \Delta S_j^{min} \qquad \text{for} \quad j = 1,\ldots,N \qquad (1.40a)$$

$$z_{N+i} = \Delta h_i - \Delta h_i^{min} \quad \text{for} \quad i = 1,\ldots,M \qquad (1.40b)$$

where, $$\Delta S_j^{min} = S_j^{min} - S_j^o \qquad (1.41a)$$

$$\Delta h_i^{min} = h_i^{min} - h_i^o \qquad (1.41b)$$

substitution of equations (1.41a) and (1.41b) in the expression for incremental objective function given by equation (1.32) yields,

$$\Delta f^K = d^T Z + d^T \begin{bmatrix} S_j^{min} \\ h_i^{min} \end{bmatrix} \qquad (1.42)$$

In equation (1.42), since second term is constant, the objective function reduces to,

$$\Delta f^K = d^T Z \qquad (1.43)$$

Then the LP problem can be stated as : Determine Z that minimizes the objective function of equation (1.43) subject to the constraints of equations (1.37) to (1.39) after substituting the values from equation (1.41a) and (1.41b).

A comparison has been made with the present work with the above mentioned dual LP technique taking a common problem, which shows that a greater accuracy has been achieved in terms of cost per unit supplied using dynamic programming technique as used in the present work (as reported in this thesis).

Singh L.P. and Aggarwal R.P.[9] used Dynamic programming technique to solve a short range problem where transmission

losses were not included to arrive at the optimal solution. A brief mathematical formulation of the above work is given below.

The problem is to determine the scheduling of generation of the hydro-thermal plants i.e. to determine $S_j$ and $h_i$ such that

$$C_T = \sum_{j=1}^{N} C_j(S_j) + \sum_{i=1}^{M} \gamma_i y_i(h_i) \tag{1.44}$$

subject to the constraints

$$\text{(a)} \quad \sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_L + P_T(S,h) \tag{1.45}$$

$$\text{(b)} \quad \left.\begin{aligned} S_j^{min} &\leq S_j \leq S_j^{max} \\ \text{and } h_i^{min} &\leq h_i \leq h_i^{max} \end{aligned}\right\} \tag{1.46}$$

$$\text{(c)} \quad \int_0^T \gamma_i y_i(h_i)dt = Y_i \tag{1.47}$$

Assuming that a total number of R plants which satisfy the total demand D, the $R^{th}$ plant supplies the demand Z. Therefore, R-1 plants will supply demand D-Z with the minimum cost of generation $F_{R-1}(D-Z)$. Hence using, principle of opti-

mality, we get,

$$F_R(D) = \min_{Z} [U_R(Z) + F_{R-1}(D-Z)$$

where, $U_R(Z)$ = cost of generating Z by the $R^{th}$ plant.

in general,

$$F_N(D) = \min_{Z} [U_N(Z) + F_{N-1}(D-Z)]$$ where, N=number of thermal plants.

To include hydel plant to run as a peak load plant, we put

R = N+1 i.e. first hydel plant is put into operation when all N thermal plants must be operating; the cost function we get,

$$F_{N+1}(D) = \min_{h_i^{min} \leq Z \leq h_i^{max}} [\gamma_1 y_1(h_1)+F_N(D-Z)]$$

of course, checking that equation (1.47) is satisfied.

OUTLINE OF THE THESIS

The objective of the present work is to obtain solutions to the complex problem of optimal scheduling in hydro-thermal

power systems including transmission losses using the technique of Dynamic programming. Merits of this scheme are increased accuracy, faster rate of convergence, and simple algorithm. A brief outline of the work reported in the different chapters is given below.

The second chapter introduces the mathematical theory in brief of the technique of dynamic programming due to R. Bellman[10] and others [11,12,13].

The third chapter describes a general approach to economic load scheduling of Hydro-thermal systems due to Kirchmayer[14], Singh L.P.[15], Stevenson Jr. W.D.[16].

In the chapter 4 the formulation and solution procedure of the problem of optimal load scheduling of hydro-thermal systems using the technique of Dynamic programming have been described. A case study comprising of three thermal and one hydel plants has been presented to illustrate the solution procedure. Transmission losses have been considered through the use of loss coefficients. A comparison with successive dual LP technique in solving a common problem has also been presented which affirms that increased accuracy, faster rate

of convergence, and simple algorithm are the assets of the present work (reported in this thesis).

Finally, chapter 5 concludes with the overall summary of the work reported in this thesis. A few proposals for further scope of investigations in this area are also made at the end.

# CHAPTER II

## DYNAMIC PROGRAMMING

### 2.1 INTRODUCTION

'Dynamic Programming' is a mathematical tool for solving problems in the catagory referred as multistage decision processes[11]. The name was so given because Dynamic Programming in many cases leads to a solution in the form of a program for a digital computer. Dynamic Programming was developed during 1950's by R. Bellman[10] to study the optimization problem arising in the industry, economics and in the social services where optimization technique of linear and non linear programming and calculus of variation and its generalization are not applicable. Such catagory of problems has been referred to as multistage decision process. Economic load scheduling of hydro-thermal system essentially belongs to the catagory of problems known as the multistage decision process requiring a sequence of decisions and which can be tackled more effectively by the technique of dynamic programming.

### 2.2 MULTISTAGE DECISION PROCESS

To understand the multistage decision process, let us consider a system which can be described at discrete times

by a finite number of variables (known as state variables) as $x_1, x_2, \ldots, x_n$. The values of these state variables at time (say stage) i is denoted by $x_1(i)$, $x_2(i)$, ..., $x_n(i)$. In the form of state vectors we can write,

$$X = [x_1, x_2, \ldots, x_n]$$

and its value at stage i is given by

$$X(i) = [x_1(i),\ x_2(i),\ \ldots\ x_n(i)]$$

Now, at stage $i = 1,2,\ldots N$, we can make a decision $d(i)$ from a number of possible choices. The effect of taking decision $d(i)$ at stage i will change the state of the system i.e. at stage i+1, the state vector is $X(i+1)$.

At stage i, of the system a return function R will be associated which is of the form,

$$R(i) = R\ [X(i),\ d(i),\ i]$$

i.e. the return function R depends upon the state of the system at the stage i.e. $X(i)$ decision $d(i)$, and stage i.

Restrictions can also be imposed on the return function to take integral values only and which can be of the form,

$$\phi\ [X(i),\ d(i),\ i] \leq \epsilon_r \quad \geq \epsilon_r \quad = \epsilon$$

Because, there are N stages and at each stage if a single decision is made then there will be sequence of decisions such as d(1), d(2),...d(N) and corresponding to each such decisions, there will be a sequence of returns. The total return I, for the whole process may be expressed as,

$$I = R(1) + R(2) + \ldots R(N)$$

or simply $$I = \sum_{i=1}^{N} R(i) \qquad (2.1)$$

The above system is called Discrete Time Multistage Decision Process.

Now we will take an example of multistage decision process. The example concerns an investor, who, each year must allocate a sum of money between two possibilities; either reinvest it or spend it. The problem is specified as follows:

(i) Let Rs X be the amount of money available in any year,

(ii) Let Rs U be the amount of money allocated to in-

vestment, so that Rs(X-U) is spent in the same year.

(iii) Let us assume that allocation decision is made only once a year and that years are numbered 1,2,...i. Then the amount of money available in year i is written as Rs X(i), the amount of money invested in year i is Rs U(i) and the amount spent in year i is Rs [X(i) - u(i)],

(iv) Assume a constant interest K (K>1) so that as a result of investing Rs u(i) in year i the amount of money available for allocation in the next year, i.e. i+1 is $x(i+1) = Ku(i)$,

(v) Let H(i) be the amount of satisfaction derived from spending money in any year i,

(vi) Assume that the satisfaction H(i) depends only on [X(i) - u(i)] i.e. the amount spent in year i, and that the relation between satisfaction and amount spent does not change with time and can be described by a known function of [X(i) - u(i)] as

$$H(i) = H[x(i) - u(i)]$$

This is called the return function,

(vii) Therefore, we can say that the total return I, derived by the investor from N consecutive years

$i = 1,\ldots N$ is given by,

$$I = H[x(1) - u(1)] + H[x(2) - u(2)] + \ldots H[x(N) - u(N)] \tag{2.2}$$

or simply $$I = \sum_{i=1}^{N} H(i) \tag{2.3}$$

Thus I depends on the initial amount of money $x(1)$ and on the series of decisions $u(1), u(2)\ldots u(N)$,

(viii) Each decision $u(i)$ is subject to restrictions because the amount of money invested in any year cannot be greater than the amount of money available that year and cannot be less than zero. i.e.

$$0 \leq u(i) \leq x(i) \; ; \; i = 1,\ldots N$$

This is an example of multistage decision process.

## 2.3 PRINCIPLE OF OPTIMALITY

The principle of optimality, orginally as stated by BELLMAN[10] is applicable to all multistage decision process and is to the effect that :

'An optimal policy has the property that, whatever be the initial state and the initial be the first decision d(1), the remaining decisions constitute an optimal policy with regard

to the state resulting from the first decision'.

Therefore, our problem is to find the optimal value of I from equation (2.1).

i.e. if the initial state is S and d(1), d(2)...d(N) decisions are taken at stages i = 1,2,...,N. Then, optimal return over N stages will be expressed as,

$$f_N(s) = \max_{d(1),d(2)\ldots d(N)} \left\{ I[S,d(1),\ldots,d(N)] \right\} \qquad (2.4)$$

## 2.4 DYNAMIC PROGRAMMING TECHNIQUE

The technique of dynamic programming depends upon the principle of optimality which is applicable to a system where the state of the system at the stage i+1 depends only upon the state of the system at the previous stage i, decision d(i) taken at stage i and the stage i itself.

The general form of the transformation in a deterministic multistage decision process satisfying the above condition is represented by N deterministic functions $G_1 \ldots G_N$ and can be written as,

$$x_1(i+1) = G_1[x_1(i), \ldots x_N(i), d(i), i]$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x_N(i+1) = G_N[x_1(i), \ldots x_N(i), d(i), i]$$

It is convenient to use vector notation by defining a vector function G having components $G_1 \ldots G_N$ i.e.

$$G = \begin{bmatrix} G_1 \\ G_2 \\ \cdot \\ \cdot \\ \cdot \\ G_N \end{bmatrix}$$

and to represent the transformation by

$$x(i+1) = G[x(i), d(i), i]$$

and the total return function can be written as

$$I = \sum_{i=1}^{N} R(i)$$

The problem is to find a sequence of N decisions, one decision at each stage for which the value of I is maximum. The same technique is precisely used in the formulation and solution of the problem envisaged in the present work which

will be described in Chapter IV later. Let the initial state of the system is s such that,

$$f_N(s) = \text{Maximum value of I when N stages remain}$$

Let us take the first decision d(1) which is arbitrary, then corresponding to this decision the state of the system becomes,

$$x(2) = G[s, d(1)]$$

and there remains N-1 stages to go. The principle of optimality states, that, when the optimal policy is used, the return from the remaining N-1 stages starting from state G[s,d(1)] is

$$f_{N-1}[G\{s, d(1)\}]$$

But the return from the first stage is,

$$R[s, d(1)]$$

So the total return is,

$$I = R[s, d(1)] + f_{N-1}[G\{s, d(1)\}] \tag{2.5}$$

When the optimal policy is used, this total return I is maximized with respect to all N decisions, including the first

decision d(1), so the optimal return over the N stage is

$$f_N(s) = \max_{d(1)} \left\{ R[S,d(1)] + f_{N-1}[G\{s,d(1)\}\ ] \right\} \quad (2.6)$$

subject to specified restrictions

or $$f_N(s) = R[s,d_N(s)] + f_{N-1}\ [G\{s,d_N(s)\}\ ] \quad (2.7)$$

Where $d_N(s)$ is the first optimal decision to be taken when the system starts with the state S and total N stages to go.

Equation (2.7) is a functional recurrence equation from which the optimal return functions $f_N(s)$ can be found. The initial solution can be found for a single-stage process, by taking N=1, when the definitions give

$$f_1(s) = \max_{d(1)}\ [R\{s,d(1)\}\ ] \quad .$$

$$= R[s,d(1)]$$

Similarly, from equation (2.7), putting N=2, gives,

$$f_2(s) = R[s,d_2(s)] + f_1\ [G\{s,d_2(s)\}\ ] \quad (2.8)$$

Equation (2.8) can be used to compute $f_2(s)$, continuing like this, we can compute $f_N(s)$ and $d_N(s)$ for any value of N . Here, $f_N(s)$ gives us the maximum value of I for the N-stage process when the initial state is s.

From the above discussion, it is evident that we have to solve a maximization (or minimization) problem in one variable only at every iteration. This is infact, the great force achieved by the use of dynamic programming. This technique has been used to solve the problem of optimal load scheduling of hydro-thermal system including transmission losses which has been brought out in chapter 4.

# CHAPTER III

# ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEMS

## 3.1 INTRODUCTION

The Economic Load Scheduling of Hydro-Thermal Systems is a complex problem and it can be solved by considering it basically a long range or a short range problem. The long range problem normally takes into account the constraints spanning over a year. The inflows into the reservoirs show an annual cyclicity. There may be seasonal variations in power demand on the system. Rainfall and head variation of reservoirs from season to season also play an important role in determining hydro-generation. Various methods have appeared in the literatures for determining economic load scheduling of Hydro-thermal systems taking long range problems into consideration. Since, the present work is a short range problem, it is beyond the scope of this chapter to explain long range problems.

The work reported here is a short range problem having an optimisation interval of day consisting of 24 hours and load demand on the system has been considered for subintervals of one hour each. The solution to the problem will stipulate the quantity of water to be utilised over a day. This quan-

tity of water must lie within the specified quantity of water over the period (a day). Therefore, the solution to the short range problem consists of an optimal plan for utilisation of this water for power generation and the corresponding optimal set of thermal generations, considering a load demand on the system over a day (24 hours), ofcourse satisfying various constraints imposed on the operation of the system.

Several attempts have been made in the past to solve a short range economic load scheduling problem of hydro-thermal systems. A brief overview of different techniques used in the past has already been given in chapter 1 with a brief introduction to their mathematical approach. However, a general approach using Lagrangian multipliers as described by Kirchmayer[14], Singh L.P.[15], Stevenson[16] is presented here in order to get acquainted with the requirements, constraints and mathematical explanation of a short range hydro-thermal system.

## 3.2 LAGRANGIAN MULTIPLIER APPROACH FOR SOLUTION OF ECONOMIC LOAD SCHEDULING OF HYDRO-THERMAL SYSTEM

The main cost of operation of hydro-thermal system is the cost of fuel for the thermal plants, because it has been assumed that the problem is of short range type and hence,

there will not be any appreciable change in the level of water in the reservoir during the interval under consideration . However, specified quantiy of water $Y_i$ must be utilised within the interval at each hydel plant i.

Thus, based upon above assumptions, the problem is to find the thermal generation $S_j$ and the hydel generation $h_i$; both of which are functions of time such that the total cost function (i.e. objective function)

$$C_T = \int_0^T \sum_{j=1}^{N} C_j(S_j)dt \tag{3.1}$$

is minimum subject to the constraints,

(a) $$\sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_L + P_T(S,h) \tag{3.2}$$

(b) $$\int_0^T y_i(h_i)dt = Y_i \tag{3.3}$$

where,

$C_j(S_j)$ = fuel cost in Rs/hr of the $j^{th}$ thermal plant

$y_i(h_i)$ = Turbine discharge in $m^3$ per hour of the $i^{th}$ hydel plant

$S$ = $S_1 S_2, \ldots, S_N$ = Generation of N thermal plants

$h = (h_1, h_2, \ldots, h_M)$ = Generation of M hydel plants

$P_L$ = Demand in MW of at any time t

$P_T$ = Transmission losses as a function of plant generation

$T$ = Final time defining the short range problem under study.

The equation (3.1) is a non linear objective function with both the constraint equations (3.2) and (3.3) being nonlinear, such problems are referred as constrained nonlinear programming problem. The nonlinear objective function with the nonlinear equality constraints can be convered into an unconstrained objective function F by proper choice of Lagranges multiplier $\lambda$ and $\gamma$

$$\text{i.e.} \quad F = C_t - \gamma_i\left(\int_0^T y_i(h_i)dt - Y_i\right) - \left(\sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i - P_L - P_T\right) \qquad (3.4)$$

Using law of calculus for minimization, the above function F can be solved by taking partial derivatives of F with respect to the plant generation S and h and equating them to zero for minimum i.e.

$$\frac{\delta F}{\delta S_j} = 0 = \frac{dC_j}{dS_j} + \lambda \frac{\delta P_T}{\delta S_j} - \lambda \tag{3.5}$$

$$\frac{\delta F}{\delta h_i} = 0 = \gamma_i \frac{dy_i}{dh_i} + \lambda \frac{\delta P_T}{\delta h_i} - \lambda \tag{3.6}$$

or,
$$\frac{dC_j}{dS_j} + \lambda \frac{\delta P_T}{\delta S_j} = \lambda \tag{3.7}$$

$$\gamma_i \frac{dy_i}{dh_i} + \lambda \frac{\delta P_T}{\delta h_i} = \lambda \tag{3.8}$$

The direct application of the above co-ordination equations (equations 3.7 and 3.8) results in solutions which sometimes dictates plant generations outside the plant capacity and also negative generations of certain plants. This phenomenon was first observed by Dandeno[2] while working with the co-ordination equations on the digital computer .

Therefore, the problem is extended by imposing upper and lower bounds on the operating range of each plant as indicated by the inequalities (equations 3.9 and 3.10)

$$S_j^{min} \leq S_j \leq S_j^{max} \quad \text{for} \quad j = 1, \ldots, N \tag{3.9}$$

$$h_i^{min} \leq h_i \leq h_i^{max} \quad \text{for} \quad i = 1, \ldots, M \tag{3.10}$$

Where $S_j^{min}$ and $h_j^{min}$ are the minimum and $S_j^{max}$ and $h_i^{max}$ are the maximum limits of operation of the corresponding plant. Dandeno also doubted about the constancy of $\gamma_i$ if these limits on the operating range of plants are included. Wije Perera[3] made an attempt to solve the problem by applying Pontryagin's maximum principle, the problem thus obtained is to maximize the Hamiltonian H at each instant of time i.e.

$$\text{Maximize} \quad H = - \sum_{j=1}^{N} C_j(S_j) - \sum_{i=1}^{M} \gamma_i y_i(h_i) \qquad (3.11)$$

The value of $\gamma_i$ should be so chosen that the condition given by the equation (3.3) is satisfied because $\gamma_i$, known as water value, actually converts the water consumption in $m^3$ per hour to the cost in Rs per hour. An increase in the value of $\gamma_i$ results in lesser water consumption at the $i^{th}$ hydel plant and vice versa. Our aim is to minimise the function given by the equation (3.11).

Since the minimization of any function is equivalent to the negative of the maximization of the same function, therefore, minimization of the cost function $C_T$ is given by

Minimise $C_T = - H$

$$= \sum_{j=1}^{N} C_j(S_j) + \sum_{i=1}^{M} \gamma_i y_i(h_i) \qquad (3.12)$$

The minimisation of the cost function $C_T$ as defined by equation (3.12) corresponds to the minimization of the total operating cost of thermal as well as hydel plants. This is because the first term in equation (3.12) represents operating cost of the thermal plant and the second term, that of hydel plant.

# CHAPTER IV

## OPTIMAL LOAD SCHEDULING OF HYDRO-THERMAL SYSTEM INCLUDING LOSSES

### 4.1 INTRODUCTION

The ultimate aim of load scheduling problem is to minimise the cost of generation of power while satisfying all the constraints imposed on the system. Several techniques including Non-linear programming, Pontryagin's maximum principle, successive dual linear programming and dynamic programming etc. were used in the past to solve a problem of short-range hydro-thermal systems [1,2,3,4,5,7,8,9]. Each has its merits and demerits.

In the present work, a short range problem of load scheduling of hydro-thermal system including transmission losses has been taken for study and solution. This chapter is devoted to the problem formulation using dynamic programming technique, its solution procedure and development of simple algorithm and finally testing it for a given test system. In a combined hydro-thermal system, almost negligible cost of generation is associated with hydro stations as the cost of water is negligible. However, there is always a constraint

in the form of availability of water (neglecting the probability of failure of plants) i.e. constraint on total energy to be generated over a day.

## 4.2 PROBLEM FORMULATION USING DYNAMIC PROGRAMMING

The optimal load scheduling of hydro-thermal system primarily belongs to the catagories of problems known as multi stage decision process and, therefore, is quite suitable for formulation and study using the technique of dynamic programming[10,11,12]. In the case of optimal load scheduling of the hydro-thermal system, we are required to take a sequence of decisions and at each decision point a number of strategies exists. The decision that we are required to take in this case, is regarding the generation schedule of different plants. Here, the stage corresponds to the number of plants that are in operation and the decision at any stage corresponds to the proper choice of generation schedule. Changing the decision at any stage will bring about a change in the state of the system. Therefore, the effect of taking decisions for generation schedule for a particular plant will automatically bring about a change in demand to be met by the remaining plants. With this change in the demand i.e. change in the state, everytime a return function in the terms of cost function of the hydro-

thermal system will be obtained; out of which one of the cost functions will surely be optimal because of taking a sequence of decisions regarding the generation schedule.

The total number of units both thermal as well as hydel available, their individual cost characteristics and the load cycle on the station are assumed to be known in advance. It is also assumed, that, the load on each unit or combination of units changes in small but uniform step size. We can start arbitrarily with any two units. The most economical combination is determined for all the discrete load levels of the combined output of the two units. At a particular load level, the most economical solution may be to run either of the units or both the units with a certain load sharing between the two units. The third unit is then added and the above procedure is repeated to find the most economic answer. The process is repeated till all available units are put into operation. The advantage of dynamic programming (DP) is that, having obtained optimal way of loading N units, it is quite easy to determine the optimal manner of loading N+1 units.

The aim of applying DP is to know the optimal scheduling of hydro-thermal plants. That is to determine $S_j$ and $h_i$ such that, total cost function $C_T$ given by,

$$C_T = \sum_{j=1}^{N} C_j(S_j) + \sum_{i=1}^{M} \gamma_i Y_i(h_i) \tag{4.1}$$

is optimal subject to the following constraints

(a) $$\sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i = P_L + P_T(S,h) \tag{4.2}$$

(b) $$S_j^{min} \leq S_j \leq S_j^{max} \tag{4.3}$$

(c) $$h_i^{min} \leq h_i \leq h_i^{max} \tag{4.4}$$

(d) $$\int_0^T y_i(h_i)dt = Y_i \tag{4.5}$$

It is obvious that the optimal value of $C_T$ depends upon the total number of stages i.e. total number of plants to be considered (M+N in this case) and the initial state of the system i.e. initial demand D which takes into account the constraint equations (4.2), (4.3) and (4.4).

Thus $$D = P_L + P_T$$

Let $F_K(D)$ = Minimum value of $C_T$ (equation 4.1) when there are K stages that is K plants in operation and the initial state is D (i.e. initial demand is D).

Let us assume that out of a total number of K plants which meet total demand D, $K^{th}$ plant meets the demand G. This is the first decision we have taken. The effect of this decision will be to meet demand D-G by the remaining K-1 plants. Let the cost of generating G by the $K^{th}$ plant be $f_K(G)$ such that,

$$f_K(G) = Th_K(G) \; ; \quad \text{if } K^{th} \text{ plant is thermal}$$

and

$$f_K(G) = \gamma_K y_K(G); \quad \text{if } K^{th} \text{ plant is hydel}$$

Therefore,

$F_{K-1}(D-G)$ = Minimum cost of generating D-G demand from K-1 plants which are in operation, ofcourse having a check that demand D-G lies between the sum of the lower bounds and the sum of the upper bounds of K-1 plants. Using the principle of optimality, we get the following relation,

$$F_K(D) = \min_{G} \; [f_K(G) + F_{K-1}(D-G)] \tag{4.6}$$

Here, it is to be noted that consequent to the initial decision that $K^{th}$ plant supplies arbitrary demand G, $F_{K-1}(D-G)$ corresponds to the remaining K-1 decisions which are always optimal with regards to the state resulting from the initial decisions i.e. first decision which is arbitrary.

According to formulation, when the first thermal plant is put into operation, we get, by putting K=1,

$$F_1(D) = \min_{G} f_1(D) = f_1(D) = C_1(D) \qquad (4.6)$$

Here, demand D must satisfy the constraint equation $S_1^{min} \leq D \leq S_1^{max}$.

To operate the second plant also, we put $K_2 = 2$ and from equation (4.6), we get,

$$F_2(D) = \underset{G}{\text{Min}}[f_2(G) + F_1(D-G)] = \underset{G}{\text{Min}}[C_2(G) + C_1(D-G)] \qquad (4.7)$$

Similarly, by putting K=3,4.... etc, we go on finding $F_3(D)$, $F_4(D)$ ...... etc till all the plants are put into operation.

While initiating hydel plant, a proper value (i.e. water value) of $\gamma_i$ is assigned and at the same time, the hydel generation also is maintained in the limits i.e. $h_i^{min} \leq h_i \leq h_i^{max}$. The value of the total discharge obtained in the case of hydel plant is compared with the specified quantity of water to be utilised by this hydel plant in the specified interval i.e.

$$\int_0^T \gamma_i y_i(h_i)dt = Y_i \qquad (4.8)$$

Initially, a low value of $\gamma$ is assigned and its value is incremented till equation (4.8) is satisfied and also demand D is met

$$\text{i.e.} \quad D \leq \left( \sum_{j=1}^{N} S_j + \sum_{i=1}^{M} h_i \right)$$

Thus, a generation schedule is obtained without including transmission losses. It is not possible to assess these losses in the beginning as the plant generation (loading) upon which these losses depend are not known in the beginning. Therefore, problem remains to obtain an optimal generation schedule including transmission losses. The same has been done in the following paragraphs. A solution procedure is also suggested.

## 4.3 SOLUTION PROCEDURE TO INCLUDE TRANSMISSION LOSSES USING DYNAMIC PROGRAMMING

Use of transmission losses as function of plant generations is made to derive the following transmission loss formula as given by Kirchmayer[14]. Here, in the loss formula, the linear term and the constant term have been excluded. And hence, transmission loss $P_L$ is given by,

$$P_L = \sum_{m=1}^{N} \sum_{n=1}^{N} P_m B_{mn} P_n \tag{4.9}$$

where, $P_L$ = Transmission loss

$P_m, P_n$ = real power generation at m, $n^{th}$ plants

$B_{mn}$ = loss coefficients also known as B coefficients which are constant under certain assumed operating conditions such as unchanged location of generators, particular transmission network and particular plants.

Equation (4.9) may be written in the matrix form as,

$$P_L = P^T B P$$

where, $P = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_N \end{bmatrix}$ and $B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{21} & B_{22} & \cdots & B_{2N} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ B_{N1} & & \cdots & B_{NN} \end{bmatrix}$

and $P^T$ is transpose of P.

Using dynamic programming, a generation schedule is obtained without including transmission losses in the first stage of optimisation. Let this schedule be $\bar{P}$. In the second stage, transmission losses using B coefficients are calculated which are given by,

$$P_L = \bar{P}^T B_{mn} \bar{P} \tag{4.10}$$

Now, let us assume that $\bar{P}$ changes slightly to include transmission losses for an optimal solution. From equation (4.10) above, the incremental transmission loss (ITL) at plant n may be expressed by,

$$\frac{\partial P_L}{\partial P_n} = 2\Sigma B_{mn} P_m \tag{4.11}$$

Let the cost of generation is $F_D$ for a particular load level D. Since, transmission losses are generally less than 15-20% of total load supplied, we can assume that individual plant generation $P_n$ changes only slightly while optimising to include losses. As a result, the per unit cost of power $F_D/D$ and the partial derivatives $\partial P_L/\partial P_n$ can reasonably be assumed to be constant in the second stage of optimisation for a particular load level.

Per unit cost of power $= F_D/D$

Change in the cost of transmission w.r.t. $P_n = (\frac{F_D}{D})(\frac{\partial P_L}{\partial P_n})$ (4.12)

Let us assume that $\bar{P} + \Delta \bar{P}$ be the optimum solution required to include transmission losses. Assuming quadratic plant cost curves and witting it for the $n^{th}$ plant as shown, we get

$$C_n = C_{0n} + C_{1n}P_n + C_{2n}P_n^2 \qquad (4.13)$$

If $P_n$ changes to $(P_n + \Delta P_n)$, then,

$$C_n + \Delta C_n = C_{0n} + C_{1n}(P_n + \Delta P_n) + C_{2n}(P_n + \Delta P_n)^2 \qquad (4.14)$$

or, $$\Delta C_n = (C_{1n} + 2P_nC_{2n})\Delta P_n + C_{2n}\,\Delta P_n^2 \qquad (4.15)$$

Now, for including cost of transmission, we will modify incremental cost function as follows,

$$\Delta C_n = (C_{1n} + 2P_nC_{2n})\Delta P_n + C_{2n}\Delta P_n^2 + (\frac{F_D}{D})(\frac{\partial P_L}{\partial P_n}).\Delta P_n \qquad (4.16)$$

$$\text{Incremental Load} = \Delta L = P_L \qquad (4.17)$$

For, the incremental lower and upper bounds, the following steps are followed,

(i) If $P_n = 0$ ; $\Delta UB_n = \Delta LB_n = 0$

where, $\Delta UB_n$ and $\Delta LB_n$ are the incremental upper and lower bounds of plant n

(ii) If $P_n \neq 0$

we consider $UB_n - P_n$ and $0.15\ P_n$ (i.e. 15% $P_n$)

$$\Delta UB_n = \text{minimum of } UB_n - P_n \text{ and } 0.15\ P_n$$

similarly, $\Delta LB_n = \text{maximum of } (-0.15\ P_n,\ LB_n - P_n)$

(iii) Having found $\Delta LB_n$ and $\Delta UB_n$ for all plants in the schedule, the following check is applied

if $\Sigma \Delta UB_n < P_L$, then step (ii) is repeated with $0.2\ P_n$ and so on till the following condition is satisfied

$$\Sigma \Delta UB_n > P_L$$

Having determined $\Delta LB_n$, $\Delta UB_n$, incremental cost function from equation (4.16) and incremental load from equation (4.17), use of dynamic programming is again made to find $\Delta P_n$.

The final generation schedule is given by $P_n + \Delta P_n$ for a load demand D.

## 4.4 CASE STUDY

A short range load scheduling problem of 24 hours duration has been considered and the period has been subdivided into interval of one hour each. The demand at the generating station is assumed to be constant during each hourly interval and at the end of each interval, the demand increases or decreases in jumps or remains constant. The system considered consists of three thermal plants and one hydel plant. The cost characteristics of thermal plants and discharge characteristics of hydel plant are given in Table I.

The transmission losses have been calculated for the given system by taking the assumed values of loss coefficients as shown in Table II.

The above problem has been solved on DEC-10 Digital Computer at IIT, KANPUR using the formulation and the solution procedure discussed in the preceeding sections for the load demand shown in Table III.

The optimal generation schedule giving optimal cost of generation, individual plant generations and transmission losses for a particular load level is shown in Table III. Solution to

the problem has been obtained for a water value ( $\lambda$ ) of 0.131. A flow chart as shown in Fig. 1 has been attached in the appendix based upon which a computer programme was developed.

A comparison has been made with the solutions obtained by the technique of successive dual linear programming. It has been found that greater accuracy has been achieved by using the technique of dynamic programming.

TABLE - I

Cost/discharge Characteristics

| Lower Bound in MW | Upper bound in MW | cost characteristics | Quantity of water to be utilized over 24 hours in $m^3$ |
|---|---|---|---|
| | | (a) Thermal plant | |
| 50 | 200 | $C_1=100+0.1\ S_1+0.01\ S_1^2$ Rs/hr | - |
| 60 | 170 | $C_2=120+0.1\ S_2+0.02\ S_2^2$ Rs/hr | - |
| 50 | 215 | $C_3=150+0.2\ S_3+0.01\ S_3^2$ Rs/hr | - |
| | | (b) Hydel plant | |
| 15 | 65 | $y_1=140+20h_i+0.06h_i^2$ cubic feet/hour | 500.00 |

TABLE - II

Loss Coefficients

| | | | |
|---|---|---|---|
| 0.0005 | 0.00005 | 0.0002 | 0.00003 |
| 0.00005 | 0.00004 | 0.00018 | -0.00011 |
| 0.0002 | 0.00018 | 0.0005 | -0.00012 |
| 0.00003 | -0.00011 | -0.00012 | 0.00023 |

TABLE III

| Interval (Each of one hour) | Demand (in MW) | Optimal (Cost in Rs.) | Generation Schedule in MW — Thermal Plant $P_1$ | $P_2$ | $P_3$ | Hydro Plant $P_4$ | Total Gen in MW | Transmission Loss in MW | Cost Per Unit Supplied (Rs/MW) | Optimal cost & per unit cost of power supplied obtained by successive dual linear programming — Optimal Cost in Rs. | Cost Per Unit Supplied (Rs/MW) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 175.0 | 557.83 | 65.0 | 59.0 | 58.0 | 0.0 | 182.0 | 7.057 | 3.1886 | 591.49 | 3.3799 |
| 2 | 190.0 | 582.72 | 73.0 | 59.0 | 67.0 | 0.0 | 199.0 | 8.858 | 3.0647 | 612.82 | 3.2257 |
| 3 | 220.0 | 641.20 | 90.0 | 59.0 | 84.0 | 0.0 | 233.0 | 13.056 | 2.9153 | 665.44 | 3.0247 |
| 4 | 280.0 | 798.87 | 121.0 | 68.0 | 114.0 | 0.0 | 303.0 | 23.135 | 2.8545 | 810.00 | 2.8929 |
| 5 | 320.0 | 930.33 | 142.0 | 74.0 | 135.0 | 0.0 | 351.0 | 31.729 | 2.9139 | 932.19 | 2.9131 |
| 6 | 360.0 | 1063.39 | 154.0 | 80.0 | 147.0 | 15.0 | 396.0 | 36.836 | 2.9608 | 1079.45 | 2.9985 |
| 7 | 390.0 | 1163.82 | 159.0 | 86.0 | 153.0 | 30.0 | 428.0 | 39.300 | 2.9941 | 1203.80 | 3.0866 |
| 8 | 410.0 | 1243.39 | 168.0 | 86.0 | 163.0 | 35.0 | 452.0 | 43.740 | 3.0456 | 1292.86 | 3.1533 |
| 9 | 440.0 | 1361.70 | 177.0 | 92.0 | 172.0 | 45.0 | 486.0 | 48.473 | 3.1123 | 1421.63 | 3.2309 |
| 10 | 475.0 | 1513.15 | 190.0 | 97.0 | 185.0 | 55.0 | 527.0 | 55.609 | 3.2100 | 1603.60 | 3.3760 |
| 11 | 525.0 | 1758.85 | 200.0 | 118.0 | 204.0 | 65.0 | 587.0 | 65.593 | 3.3733 | 1842.16 | 3.5089 |
| 12 | 550.0 | 1917.18 | 200.0 | 139.0 | 215.0 | 65.0 | 619.0 | 71.034 | 3.4987 | 1929.23 | 3.5077 |
| 13 | 565.0 | 2038.38 | 200.0 | 159.0 | 215.0 | 65.0 | 639.0 | 72.934 | 3.6010 | 2080.14 | 3.6817 |
| 14 | 540.0 | 1852.14 | 200.0 | 127.0 | 215.0 | 65.0 | 607.0 | 69.909 | 3.4485 | 1914.92 | 3.5461 |
| 15 | 500.0 | 1620.86 | 196.0 | 103.0 | 191.0 | 65.0 | 555.0 | 59.232 | 3.2694 | 1703.63 | 3.4072 |
| 16 | 450.0 | 1409.87 | 184.0 | 92.0 | 178.0 | 45.0 | 499.0 | 51.927 | 3.1536 | 1471.61 | 3.2689 |
| 17 | 425.0 | 1298.81 | 171.0 | 92.0 | 165.0 | 40.0 | 468.0 | 45.280 | 3.0725 | 1347.01 | 3.1694 |
| 18 | 400.0 | 1207.19 | 166.0 | 86.0 | 159.0 | 30.0 | 441.0 | 42.416 | 3.0287 | 1248.44 | 3.1211 |
| 19 | 375.0 | 1116.49 | 160.0 | 80.0 | 153.0 | 20.0 | 413.0 | 39.437 | 2.9888 | 1138.74 | 3.0366 |
| 20 | 340.0 | 989.03 | 144.0 | 74.0 | 139.0 | 15.0 | 372.0 | 32.459 | 2.9129 | 1003.46 | 2.9513 |
| 21 | 300.0 | 861.83 | 132.0 | 69.0 | 126.0 | 0.0 | 327.0 | 27.534 | 2.8779 | 870.63 | 2.9021 |
| 22 | 250.0 | 713.72 | 107.0 | 60.0 | 101.0 | 0.0 | 268.0 | 18.115 | 2.8562 | 730.17 | 2.9207 |
| 23 | 200.0 | 600.39 | 79.0 | 59.0 | 72.0 | 0.0 | 210.0 | 10.122 | 3.0038 | 628.61 | 3.1430 |
| 24 | 180.0 | 566.29 | 68.0 | 59.0 | 61.0 | 0.0 | 188.0 | 7.668 | 3.1403 | 598.31 | 3.3239 |

Total DISCHARGE of water = 498.974 $m^3$ for water value ($\gamma$) = 0.131
Total System cost of Generation = Rs. 27807.43

Total discharge of water = 499.25 $m^3$
Total system cost of generation = Rs.28770.36

TABLE III

| Interval (Each of one hour) | Demand (in MW) | Optimal (Cost in Rs.) | Generation Schedule in MW: Thermal Plant $P_1$ | $P_2$ | $P_3$ | Hydro Plant $P_4$ | Total Gen in MW | Transmission Loss in MW | Cost Per Unit Supplied (Rs/MW) | Optimal cost & per unit cost of power supplied obtained by successive dual linear programming: Optimal Cost in Rs. | Cost Per Unit Supplied (Rs/MW) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 175.0 | 557.83 | 65.0 | 59.0 | 58.0 | 0.0 | 182.0 | 7.057 | 3.1886 | 591.49 | 3.3799 |
| 2 | 190.0 | 582.72 | 73.0 | 59.0 | 67.0 | 0.0 | 199.0 | 8.858 | 3.0647 | 612.82 | 3.2257 |
| 3 | 220.0 | 641.20 | 90.0 | 59.0 | 84.0 | 0.0 | 233.0 | 13.056 | 2.9153 | 665.44 | 3.0247 |
| 4 | 280.0 | 798.87 | 121.0 | 68.0 | 114.0 | 0.0 | 303.0 | 23.135 | 2.8545 | 810.00 | 2.8929 |
| 5 | 320.0 | 930.33 | 142.0 | 74.0 | 135.0 | 0.0 | 351.0 | 31.729 | 2.9139 | 932.19 | 2.9131 |
| 6 | 360.0 | 1063.39 | 154.0 | 80.0 | 147.0 | 15.0 | 396.0 | 36.836 | 2.9608 | 1079.45 | 2.9985 |
| 7 | 390.0 | 1163.82 | 159.0 | 86.0 | 153.0 | 30.0 | 428.0 | 39.300 | 2.9941 | 1203.80 | 3.0866 |
| 8 | 410.0 | 1243.39 | 168.0 | 86.0 | 163.0 | 35.0 | 452.0 | 43.740 | 3.0456 | 1292.86 | 3.1533 |
| 9 | 440.0 | 1361.70 | 177.0 | 92.0 | 172.0 | 45.0 | 486.0 | 48.473 | 3.1123 | 1421.63 | 3.2309 |
| 10 | 475.0 | 1513.15 | 190.0 | 97.0 | 185.0 | 55.0 | 527.0 | 55.609 | 3.2100 | 1603.60 | 3.3760 |
| 11 | 525.0 | 1758.85 | 200.0 | 118.0 | 204.0 | 65.0 | 587.0 | 65.593 | 3.3733 | 1842.16 | 3.5089 |
| 12 | 550.0 | 1917.18 | 200.0 | 139.0 | 215.0 | 65.0 | 619.0 | 71.034 | 3.4987 | 1929.23 | 3.5077 |
| 13 | 565.0 | 2038.38 | 200.0 | 159.0 | 215.0 | 65.0 | 639.0 | 72.934 | 3.6010 | 2080.14 | 3.6817 |
| 14 | 540.0 | 1852.14 | 200.0 | 127.0 | 215.0 | 65.0 | 607.0 | 69.909 | 3.4485 | 1914.92 | 3.5461 |
| 15 | 500.0 | 1620.86 | 196.0 | 103.0 | 191.0 | 65.0 | 555.0 | 59.232 | 3.2694 | 1703.63 | 3.4072 |
| 16 | 450.0 | 1409.87 | 184.0 | 92.0 | 178.0 | 45.0 | 499.0 | 51.927 | 3.1536 | 1471.61 | 3.2689 |
| 17 | 425.0 | 1298.81 | 171.0 | 92.0 | 165.0 | 40.0 | 468.0 | 45.280 | 3.0725 | 1347.01 | 3.1694 |
| 18 | 400.0 | 1207.19 | 166.0 | 86.0 | 159.0 | 30.0 | 441.0 | 42.416 | 3.0287 | 1248.44 | 3.1211 |
| 19 | 375.0 | 1116.49 | 160.0 | 80.0 | 153.0 | 20.0 | 413.0 | 39.437 | 2.9888 | 1138.74 | 3.0366 |
| 20 | 340.0 | 989.03 | 144.0 | 74.0 | 139.0 | 15.0 | 372.0 | 32.459 | 2.9129 | 1003.46 | 2.9513 |
| 21 | 300.0 | 861.83 | 132.0 | 69.0 | 126.0 | 0.0 | 327.0 | 27.534 | 2.8779 | 870.63 | 2.9021 |
| 22 | 250.0 | 713.72 | 107.0 | 60.0 | 101.0 | 0.0 | 268.0 | 18.115 | 2.8562 | 730.17 | 2.9207 |
| 23 | 200.0 | 600.39 | 79.0 | 59.0 | 72.0 | 0.0 | 210.0 | 10.122 | 3.0038 | 628.61 | 3.1430 |
| 24 | 180.0 | 566.29 | 68.0 | 59.0 | 61.0 | 0.0 | 188.0 | 7.668 | 3.1403 | 598.31 | 3.3239 |

Total DISCHARGE of water = 498.974 $m^3$ for water value ($\gamma$) = 0.131
Total System cost of Generation = Rs. 27807.43

Total discharge of water = 499.25 $m^3$
Total system cost of generation = Rs.28770.36

# CHAPTER V

## CONCLUSION

In this chapter a broad review of the work reported in this thesis and a few proposals for the scope of future investigations in this area have been presented.

The optimal load scheduling of hydro-thermal system is a complex problem. Several attempts were made in the past to solve such problem including nonlinear programming techniques, maximum principle of Pontryagin and successive dual linear programming. Each has its own merits and demerits. Since, economic load scheduling of hydro-thermal systems belongs to the category of problem known as multistage decision process, the technique of dynamic programming which is an appropriate mathematical tool for such categories of problems has been used in the present work for a realistic optimal solution. In this formulation, transmission losses were also included by the use of loss-coefficients. Load demands spanning over a period of 24 hours have been taken into consideration and these have been treated as deterministic. A case study has been presented by taking a short range scheduling problem having three thermal and one hydel plant. The algorithm developed is very simple and it does not require any complex

mathematical approach. This is infact an edge over various other techniques used in the past to tackle a short-range problem. Choice of water value $\gamma$ is very important to solve the problems of this nature. However, by incorporating Binary search in the present work to find $\gamma_i$, a considerable reduction in solution time is achieved even with flat start of $\gamma_i^{max}$ and $\gamma_i^{min}$ say 1.0 and 0.0 respectively in the beginning. The solution to the present problem was found for a water value of 0.131. If we start from $\gamma = 0.0$ in the beginning and adopt sequential search, we have to go through as many as 131 iterations to come to the solution of the problem. But by using Binary search we have to carry out only $\log_2 131$ i.e. about 8 iterations to arrive at the final solution which is certainly a great improvement over sequential search. The final generation schedule is more realistic and optimal cost is found to be more accurate and acceptable. A comparison has been made with the solutions obtained by the technique of successive dual linear programming. The proposed method is very effective for a system which is predominantly thermal with a few hydel stations. In case the number of hydel plants is more, they can be replaced by an equivalent hydel plant which will have the total energy equal to the sum of the energy of the respective hydel plants provided these plants are located at about the same distance from the load centre.

## SCOPE OF FUTURE INVESTIGATIONS IN THE AREA

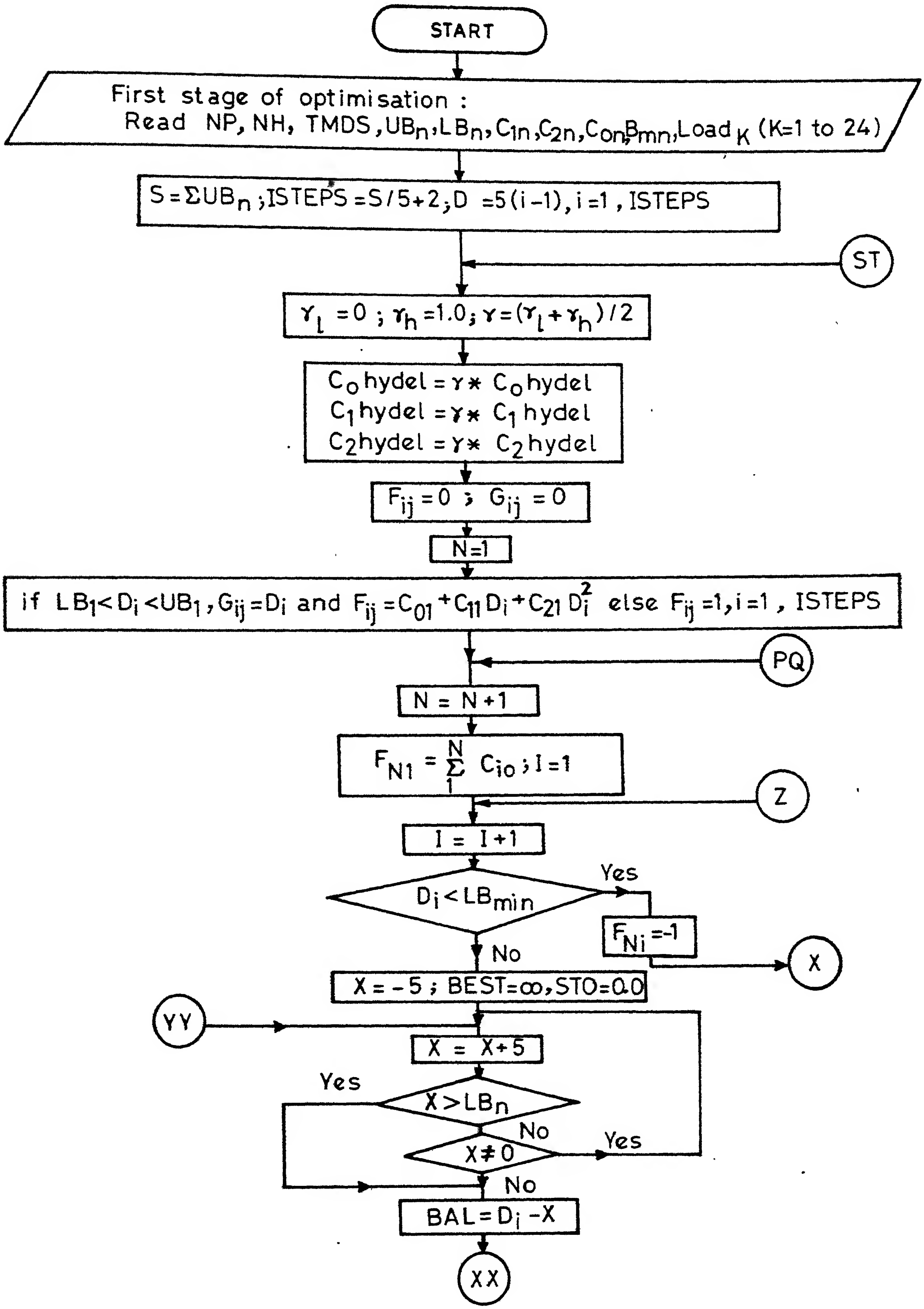
Further investigations in the area are proposed consequent to the achievements made in the present work. They are as follows:

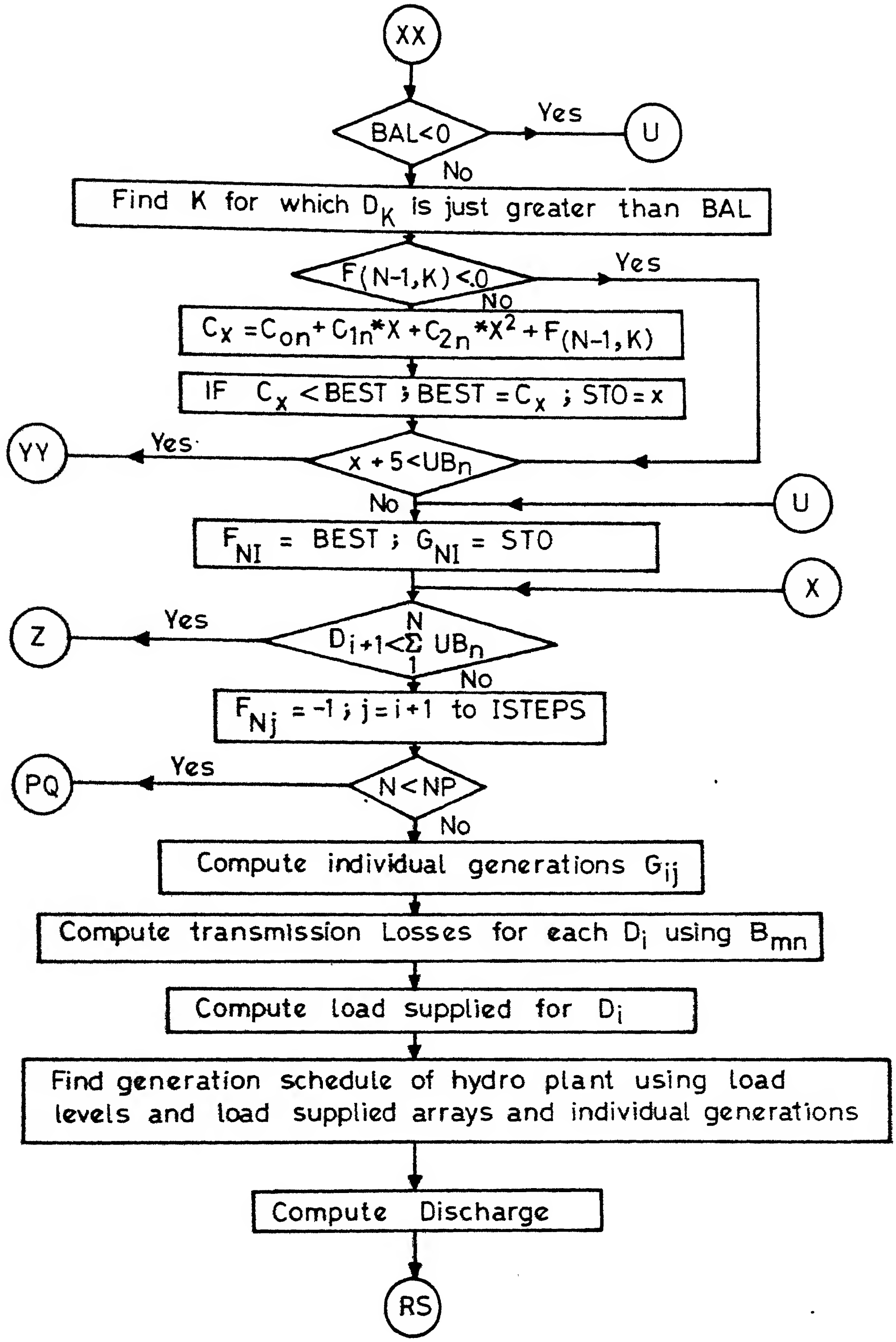
1. In the above problem, a deterministic load demand has been assumed but in practice, there may be uncertainties in the load demand i.e. making the problem a probabilistic one. Also discharge of water may be uncertain over the period under study. It is, therefore, proposed to make suitable modifications of the solution procedure to take into account these uncertainties.

2. In the formulation of the problem, if reactive power also is taken into consideration, further savings can be achieved. Because, both real and reactive power contribute to the transmission losses. In that case, the linear and the constant term of the loss formula also are to be taken into account. Use of load flow analysis could be made to determine reactive powers and the line flows and they can be included in the problem as additional constraints. But at the same time, it is

quite obvious that computational time will increase and therefore, it would be important to chalk out a balance between the computational efforts and time required and the optimal answers obtained. Although, according to Kirchmayer[14] it was found that the additional savings obtained by the use of loss-formula with linear and constant terms included over those obtained by average loss formula excluding the linear and constant term, are marginal.

# APPENDIX


START
First stage of optimisation :
Read NP, NH, TMDS, UBn, LBn, C1n, C2n, Con, Bmn, Load K (K=1 to 24)
S = ΣUBn ; ISTEPS = S/5+2 ; D = 5(i-1), i=1, ISTEPS
ST
γl = 0 ; γh = 1.0 ; γ = (γl + γh)/2
Co hydel = γ* Co hydel
C1 hydel = γ* C1 hydel
C2 hydel = γ* C2 hydel
Fij = 0 ; Gij = 0
N=1
if LB1 < Di < UB1, Gij = Di and Fij = C01 + C11 Di + C21 Di2 else Fij = 1, i=1, ISTEPS
PQ
N = N+1
FN1 = Σ Cio ; I=1
Z
I = I+1
Di < LBmin
Yes
FNi = -1
X
No
X = -5 ; BEST=∞, STO=0.0
YY
X = X+5
Yes
X > LBn
No
X≠0
Yes
No
BAL = Di - X
XX


XX
BAL<0
Yes
U
No
Find K for which $D_K$ is just greater than BAL
F(N-1,K)<0
Yes
No
$C_x = C_{on} + C_{1n}*X + C_{2n}*X^2 + F(N-1,K)$
IF $C_x$ < BEST ; BEST = $C_x$ ; STO = x
YY
Yes
x + 5 < $UB_n$
No
U
$F_{NI}$ = BEST ; $G_{NI}$ = STO
X
Z
Yes
$D_{i+1} < \sum_1^N UB_n$
No
$F_{Nj}$ = -1 ; j = i+1 to ISTEPS
PQ
Yes
N < NP
No
Compute individual generations $G_{ij}$
Compute transmission Losses for each $D_i$ using $B_{mn}$
Compute load supplied for $D_i$
Find generation schedule of hydro plant using load levels and load supplied arrays and individual generations
Compute Discharge
RS

MEANINGS OF NOTATIONS AS USED IN THE FLOW CHART OTHER THAN THOSE ALREADY USED IN THE TEXT

| | | |
|---|---|---|
| NP | = | Total number of plants |
| NH | = | Number of hydel plants |
| MDS | = | Minimum discharge specified |
| $UB_n$ | = | Upper bound of $n^{th}$ plant |
| $LB_n$ | = | Lower bound of $n^{th}$ plant |
| $C_{01}, C_{1n}, C_{2n}$ | = | Cost curve coefficients of $n^{th}$ plant |
| $B_{mn}$ | = | Loss coefficient B-matrix |
| $Load_K$ | = | $K^{th}$ load level |
| $D_i$ | = | $i^{th}$ demand in the demand array |
| $F_{ij}$ | = | Cost of generation of $j^{th}$ plant at $i^{th}$ demand level |
| BEST | = | Best solution so far obtained |
| $LB_{mn}$ | = | minimum lower bound |
| $P_{LK}$ | = | transmission loss at $K^{th}$ load level |
| Epsillon | = | very small number |
| $\gamma_l$ | = | lowest value of $\gamma$ (water value) say 0.0 at start |
| $\gamma_h$ | = | highest value of $\gamma$ say 1.0 at start. |

# REFERENCES


[1] W.G. Chandler, P.L. Dandeno, A.F. Glimn and L.K. Kirchmayer, 'Short-Range Economic Operation of a Combined Thermal and Hydro-Electric Power System', AIEE Trans, Vol.72, pp. 1057-1065, Oct. 1953.

[2] P.L. Dandeno, 'Hydro-Thermal Economic Load Scheduling Computation experience with the Co-ordination Equations', AIEE Trans, vol.80, pp. 1219-1228, May 1961.

[3] K.K.Y. Wije Perera, G.F. Schrack and Frank Noakes, 'Short-Range Economic Optimisation of Hydro-Thermal Power System Considering Operating Range Limitations and Unit Commitment Aspects', Presented at the IEEE summer Power Meeting, June 22-27, 1969.

[4] B.G. Sokkappa, 'Optimal Scheduling of Hydro-Thermal Systems - A Generalised Approach', IEEE Trans, vol. PAS 82, April 1963, pp. 97-104.

[5] J. Gopala Rao, 'Optimal Load Scheduling of Power Systems Using Non-Linear Programming Techniques', Ph.D. Thesis, IIT, Kanpur, July 1970.

[6] L.S. Lason, 'Duality and Decomposition in Mathematical Programming', IEEE Trans, SSC, vol.4, 1968, pp. 86-100.

[7] I. Hano, Y. Tamura and S. Narita, 'Application of the Maximum Principle to the Most Economic Operation of Power Systems', IEEE Trans, PAS, vol. 85, 1966, pp.486-494.

[8] K.N. Srivastava and L.P. Singh, 'Economic Load Scheduling of Hydro-Thermal System Including Transmission Losses', Journal of Institution of Engineers (India), pp. 121-126, vol. 67, part EL3, December 1986.

[9] L.P. Singh and R.P. Aggarwal, 'Economic Load Scheduling of Hydro-Thermal Stations Using Dynamic Programming', Journal of Institution of Engineers (India) pp. 175-180, vol.52, No.8, Part EL-4, April 1972.

[10] R. Bellman, 'Dynamic Programming', Princeton, New Jersey, Princeton University Press, 1957.

[11] O.L.R. Jacobs, 'An Introduction to Dynamic Programming', (book). Chapman and Hall Ltd., 1967.

[12] R. Bellman,'Adaptive Control Process : A Guided Tour', (book) Princeton, New Jersey, Princeton University Press, 1961.

[13] L.S. Pontryagin, 'The Mathematical Theory of Optimal Process (English Translation)',(book) New York, Interscience, 1962.

[14] L.K. Kirchmayer, 'Economic Operation of Power Systems', (book) Wiley Eastern Limited, First reprint 1979.

[15] L.P. Singh, 'Advanced Power Systems Analysis and Dynamics', (book) Wiley Eastern Limited, 1982.

[16] William D. Stevenson, Jr. 'Elements of Power System Analysis', (book) McGraw-Hill International Book Company, Fourth Edition, 1985.

[17] S. Lipschutz, A. Poe, 'Programming with Fortran', (book), Schaum's Outline series, McGraw-Hill International Book Company, First Impression, June 1982.

[18] H.W. Dommel and W.F. Tinney, 'Optimal Power Flow Solutions', IEEE Trans. PAS, vol. 87, No.10, Oct.1968, pp. 225-239.